Prolegomena to General-Imaging-Based Probabilistic Dynamic Epistemic Logic*

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Abstract. In this paper, I propose a new version of probabilistic dynamic epistemic logic (GIPDEL) that is based on general imaging, and sketch the proof of soundness and completeness of this logic. The Monty Hall dilemma is a common topic in probabilistic dynamic epistemic logic. Using product-update-rule-based probabilistic dynamic epistemic logic (PUPDEL), Kooi ([7]) supported the answer that I should switch my choice. However, it is acknowledged that this answer is counterintuitive. Using GIPDEL, I can support the answer that I do not have to switch my choice. Intuition would suggest this answer. Moreover, GIPDEL can give a plausible answer to a modified version of the Monty Hall dilemma to which PUPDEL gives an extremely counterintuitive answer.

1 Introduction

Epistemic logic is the logic of knowledge. Dynamic epistemic logic is an extension of epistemic logic that can be used to reason about knowledge changes. Kooi combined probability with dynamic epistemic logic.[1] Because this logic is based on product update rule, I call it product-update-rule-based probabilistic dynamic epistemic logic (PUPDEL). The Monty Hall dilemma is an open problem which is well-known among linguists, philosophers, psychologists, and logicians. This dilemma has the same structure as the problem of three prisoners. Nowadays this dilemma is a common topic in probabilistic dynamic epistemic logic. In [7], using PUPDEL, Kooi supported the answer that I should switch my choice. However, it is acknowledged that this answer is counterintuitive.[2] Imaging is a method of changing probability functions Lewis proposed in [9]. Gärdenfors generalised this method in [3]. In this paper, I propose a new version of probabilistic dynamic epistemic logic that is based on general imaging and sketch the proof of

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[1] [7].

[2] For the view that this answer is counterintuitive, refer to [5].

soundness and completeness of this logic. I call this logic general-imaging-based probabilistic dynamic epistemic logic (GIPDEL). Using GIPDEL, I can support the answer that I do not have to switch my choice. Intuition would suggest this answer. Moreover, GIPDEL can give a plausible answer to a modified version of the Monty Hall dilemma to which PUPDEL gives an extremely counterintuitive answer.

2 Probabilistic Epistemic Logic PEL

2.1 Language

Fagin and Halpern gave the language of PEL $\mathcal{L}_{\mathsf{PEL}}$.[3]

Definition 1. *$\mathcal{L}_{\mathsf{PEL}}$ is defined in terms of a countable set S of sentential variables, a finite set A of agents, an epistemic operator $\mathbf{K}_a$ and a probability function symbol $\mathbf{P}_a$. The well-formed formulae of $\mathcal{L}_{\mathsf{PEL}}$ are given by the following rule:*

$$\phi ::= s \mid \top \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \mathbf{K}_a(\phi) \mid \sum_{i=1}^{n} r_i \mathbf{P}_a(\phi_i) \geq r,$$

where $s \in \mathcal{S}, a \in \mathcal{A}$ and $r_1, \ldots, r_n, r \in \mathbb{Q}$. $\sum_{i=1}^{n} r_i \mathbf{P}_a(\phi_i)$ is called a term of $\mathcal{L}_{\mathsf{PEL}}$, and $\sum_{i=1}^{n} r_i \mathbf{P}_a(\phi_i) \geq r$ is called an a-probability formula of $\mathcal{L}_{\mathsf{PEL}}$. Let Φ_{PEL} denote the set of all well-formed formulae of $\mathcal{L}_{\mathsf{PEL}}$. Let $P_{a,\mathcal{L}_{\mathsf{PEL}}}$ denote the set of all a-probability formulae of $\mathcal{L}_{\mathsf{PEL}}$ and let $T_{\mathcal{L}_{\mathsf{PEL}}}$ denote the set of all terms of $\mathcal{L}_{\mathsf{PEL}}$.

$\bot, \vee, \rightarrow$ and $\leftrightarrow$ are introduced by the standard definitions. We use the usual abbreviations for readability.

2.2 Semantics

Fagin and Halpern defined a multi-agent structured Kripke model for PEL as follows:[4]

Definition 2. *A multi-agent structured Kripke model for PEL $\mathcal{M}_{\mathsf{PEL}}$ is an $(n+3)$-tuple $(W, \pi, R_{a_1}, \ldots, R_{a_n}, \mathcal{P})$, where W is a set of possible worlds, π is a truth assignment to each $s \in S$ for each $w \in W$, R_{a_i} is an equivalence relation on $W \times W$ for $i = 1, \ldots, n$, and $\mathcal{P}$ is a probability assignment that assigns to each $a \in A$ and each $w \in W$ a probability space $\mathcal{P}(a, w) = (W_{a,w}, \mathcal{F}_{a,w}, P_{a,w})$, where $W_{a,w} \subset W$ is the sample space, $\mathcal{F}_{a,w}$ is a σ-field of subsets of $W_{a,w}$, and $P_{a,w}$ is a probability measure defined on $\mathcal{F}_{a,w}$. We define $R_a(w_1)$ and $W_{a,w_1}(\phi)$ as follows:*

$$R_a(w_1) := \{w_2 : (w_1, w_2) \in R_a\},$$
$$W_{a,w_1}(\phi) := \{w_2 \in W_{a,w_1} : (\mathcal{M}_{\mathsf{PEL}}, w_2) \models \phi\}.$$

[3] [[1]: 343].
[4] [[1]: 343, 346].

Moreover, $\mathcal{M}_{\mathsf{PEL}}$ satisfies the following conditions:[5]

$$
\begin{array}{ll}
(\mathit{CONS}) & \begin{array}{l} \text{For all } a \in A \text{ and } w \in W, \\ \text{if } \mathcal{P}(a,w) = (W_{a,w}, \mathcal{F}_{a,w}, P_{a,w}), \text{ then } W_{a,w} \subset R_a(w), \end{array} \\
(\mathit{OBJ}) & \mathcal{P}(a_1,w) = \mathcal{P}(a_2,w) \text{ for all } a_1, a_2 \in A \text{ and } w \in W, \\
(\mathit{SDP}) & \begin{array}{l} \text{For all } a \in A \text{ and } w_1, w_2 \in W, \\ \text{if } w_2 \in R_a(w_1), \text{ then } \mathcal{P}(a,w_1) = \mathcal{P}(a,w_2), \end{array} \\
(\mathit{UNIF}) & \begin{array}{l} \text{For all } a \in A \text{ and } w_1, w_2 \in W \\ \text{if } \mathcal{P}(a,w_1) = (W_{a,w_1}, \mathcal{F}_{a,w_1}, P_{a,w_1}) \text{ and } w_2 \in W_{a,w_1}, \\ \text{then } \mathcal{P}(a,w_2) = \mathcal{P}(a,w_1), \end{array} \\
(\mathit{MEAS}) & \begin{array}{l} \text{For all } a \in A \text{ and } w \in W \text{ and } \phi \in \Phi_{\mathcal{L}_{\mathsf{PEL}}}, \\ W_{a,w}(\phi) \in \mathcal{F}_{a,w}, \end{array}
\end{array}
$$

Let $\mathbb{M}_{\mathsf{PEL}}$ denote the class of all structured Kripke models for PEL.

(CONS) postulates that the belief system of an agent who places positive probability on an event he knows to be false is inconsistent. (OBJ) postulates the objectivity of probability assignments. (SDP) postulates that the choice of probability space is the same in all worlds the agent considers possible. (UNIF) postulates that we can partition $R_a(w)$ into subsets such that at every world in a given subset, the probability space is the same. (MEAS) postulates that all well-formed formulae define measurable sets.

Fagin and Halpern gave the following truth definition.[6]

Definition 3. *The notion of $\phi \in \Phi_{\mathcal{L}_{\mathsf{PEL}}}$ being true at $w \in W$ in $\mathcal{M}_{\mathsf{PEL}}$, in symbols $(\mathcal{M}_{\mathsf{PEL}}, w) \models \phi$ is inductively defined as follows:*

$$
\begin{array}{l}
(\mathcal{M}_{\mathsf{PEL}}, w) \models s \quad \text{iff} \quad \pi(w)(s) = \mathbf{true}, \\
(\mathcal{M}_{\mathsf{PEL}}, w) \models \phi_1 \wedge \phi_2 \quad \text{iff} \quad (\mathcal{M}_{\mathsf{PEL}}, w) \models \phi_1 \quad \text{and} \quad (\mathcal{M}_{\mathsf{PEL}}, w) \models \phi_2, \\
(\mathcal{M}_{\mathsf{PEL}}, w) \models \neg\phi \quad \text{iff} \quad (\mathcal{M}_{\mathsf{PEL}}, w) \not\models \phi, \\
(\mathcal{M}_{\mathsf{PEL}}, w_1) \models K_a(\phi) \quad \text{iff} \quad (\mathcal{M}_{\mathsf{PEL}}, w_2) \models \phi \quad \text{for all } w_2 \in R_a(w_1), \\
(\mathcal{M}_{\mathsf{PEL}}, w) \models \sum_{i=1}^{n} r_i \mathbf{P}_a(\phi_i) \geq r \quad \text{iff} \quad \sum_{i=1}^{n} r_i P_{a,w}(W_{a,w}(\phi_i)) \geq r.
\end{array}
$$

If $(\mathcal{M}_{\mathsf{PEL}}, w) \models \phi$ for all $w \in W$, we write $\mathcal{M} \models \phi$ and say that ϕ is valid in $\mathcal{M}$. If ϕ is valid in all models in $\mathbb{M}_{\mathsf{PEL}}$, we write $\mathbb{M}_{\mathsf{PEL}} \models \phi$ and say that ϕ is valid with respect to $\mathbb{M}_{\mathsf{PEL}}$.

We define the probability of the semantic value of $\phi \in \Phi_{\mathcal{L}_{\mathsf{PEL}}}$ as follows:

Definition 4.

$$
P_{a,w_1}(W_{a,w_1}(\phi)) := \begin{cases} \displaystyle\sum_{w_2 \in W_{a,w_1}(\phi)} P_{a,w_1}(\{w_2\}) & \text{if } \vdash_{\mathsf{PEL}} \phi \not\leftrightarrow \bot, \\ 0 & \text{otherwise.} \end{cases}
$$

[5] [[1]: 350–352].
[6] [[1]: 343, 347].

2.3 Syntax

Fagin and Halpern gave the axiom system of PEL as follows:[7]

Definition 5.

- ***Axioms of*** PEL

(A1) *All tautologies of classical sentential logic,*

(A2) $\mathbf{K}_a(\phi_1 \to \phi_2) \to (\mathbf{K}_a(\phi_1) \to \mathbf{K}_a(\phi_2))$ (K),

(A3) $\mathbf{K}_a(\phi) \to \phi$ (T),

(A4) $\mathbf{K}_a(\phi) \to \mathbf{K}_a\mathbf{K}_a(\phi)$ (*Positive Introspection*),

(A5) $\neg\mathbf{K}_a(\phi) \to \mathbf{K}_a\neg\mathbf{K}_a(\phi)$ (*Negative Introspection*),

(A6) $\mathbf{P}_a(\phi) \geq 0$ (*Nonnegativity*),

(A7) $\mathbf{P}_a(\top) = 1$ (*Normalisation*),

(A8) $\mathbf{P}_a(\phi_1 \wedge \phi_2) + \mathbf{P}_a(\phi_1 \wedge \neg\phi_2) = \mathbf{P}_a(\phi_1)$ (*Additivity*),

(A9) $(\sum_{i=1}^{n} r_i\mathbf{P}_a(\phi_i) \geq r) \leftrightarrow (\sum_{i=1}^{n} r_i\mathbf{P}_a(\phi_i) + 0\mathbf{P}_a(\phi_{n+1}) \geq r)$
(Adding and Deleting 0 Terms),

(A10) $(\sum_{i=1}^{n} r_i\mathbf{P}_a(\phi_i) \geq r) \to (\sum_{i=1}^{n} r_{j_i}\mathbf{P}_a(\phi_{j_i}) \geq r)$
if $j_1, \ldots, j_n$ *is a permutation of* $1, \ldots, n$ *(Permutation),*

(A11) $(\sum_{i=1}^{n} r_i\mathbf{P}_a(\phi_i) \geq r) \wedge (\sum_{i=1}^{n} r'_i\mathbf{P}_a(\phi_i) \geq r') \to \sum_{i=1}^{n}(r_i + r'_i)\mathbf{P}_a(\phi_i) \geq (r + r')$
(Addition of Coefficients),

(A12) $(\sum_{i=1}^{n} r_i\mathbf{P}_a(\phi_i) \geq r) \leftrightarrow (\sum_{i=1}^{n} r' r_i\mathbf{P}_a(\phi_i) \geq r' r)$
if $r' > 0$ *(Multiplication of Nonzero Coefficients),*

(A13) $(t \geq r) \vee (t \leq r)$ *if* $t \in T_{\mathcal{L}_{\mathsf{PEL}}}$ *(Dichotomy),*

(A14) $(t \geq r_1) \to (t > r_2)$ *if* $t \in T_{\mathcal{L}_{\mathsf{PEL}}}$ *and* $r_1 > r_2$ *(Monotonicity),*

(A15) $\mathbf{K}_a(\phi) \to (\mathbf{P}_a(\phi) = 1)$ *(Correspondent to CONS),*

(A16) $(\sum_{i=1}^{n} r_i\mathbf{P}_a(\phi_i) \geq r) \to (\sum_{i=1}^{n} r_{j_i}\mathbf{P}_b(\phi_{j_i}) \geq r)$ *(Correspondent to OBJ),*

(A17) $\phi_1 \to (\mathbf{P}_a(\phi_1) = 1)$ *if* $\phi_1 \in P_{a,\mathcal{L}_{\mathsf{PEL}}}$ *or* ϕ_1 *is* $\neg\phi_2$ *such that* $\phi_2 \in P_{a,\mathcal{L}_{\mathsf{PEL}}}$
(Correspondent to UNIF),

(A18) $\phi_1 \to \mathbf{K}_a(\phi_1)$ *if* $\phi_1 \in P_{a,\mathcal{L}_{\mathsf{PEL}}}$ *or* ϕ_1 *is* $\neg\phi_2$ *such that* $\phi_2 \in P_{a,\mathcal{L}_{\mathsf{PEL}}}$
(Correspondent to SDP),

- ***Inference Rules of*** PEL

(R1) $\dfrac{\phi_1 \quad \phi_1 \to \phi_2}{\phi_2}$ (*Modus Ponens*),

(R2) $\dfrac{\phi}{\mathbf{K}_a(\phi)}$ (*Generalisation*),

(R3) $\dfrac{\phi_1 \leftrightarrow \phi_2}{\mathbf{P}_a(\phi_1) = \mathbf{P}_a(\phi_2)}$ (*Distributivity*).

[7] [[1]: 344, 353, 357].

If $\phi \in \Phi_{\mathsf{PEL}}$ is provable by (R1),(R2) or (R3) from (A1),(A2),(A3),(A4),(A5),(A6),(A7),(A8),(A9),(A10),(A11),(A12),(A13),(A14),(A15),(A16),(A17) or (A18), we write $\vdash_{\mathsf{PEL}} \phi$. The subsystem consisting of (A1),(A2),(A3),(A4),(A5),(R1) and (R2) is called S5.

2.4 Soundness and Completeness

We can prove the soundness theorem of PEL in the usual way.

Theorem 1. ***(Soundness)***
If $\vdash_{\mathsf{PEL}} \phi$, then $\mathbb{M}_{\mathsf{PEL}} \models \phi$.

Fagin and Halpern proved the completeness theorem of PEL.[8]

Theorem 2. ***(Completeness)***
If $\mathbb{M}_{\mathsf{PEL}} \models \phi$, then $\vdash_{\mathsf{PEL}} \phi$.

3 Product-Update-Rule-Based Probabilistic Dynamic Epistemic Logic PUPDEL

3.1 Language

Kooi gave the language of PUPDEL $\mathcal{L}_{\mathsf{PUPDEL}}$.[9]

Definition 6. *$\mathcal{L}_{\mathsf{PUPDEL}}$ is defined in terms of a countable set S of sentential variables, a finite set A of agents, an epistemic operator $\mathbf{K}_a$, a probability function symbol $\mathbf{P}_a$ and an update operator $[\ ]$. The well-formed formulae of $\mathcal{L}_{\mathsf{PUPDEL}}$ are given by the following rule:*

$$\phi ::= s \mid \top \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \mathbf{K}_a(\phi) \mid \sum_{i=1}^{n} r_i \mathbf{P}_a(\phi_i) \geq r \mid [\phi_1]\phi_2,$$

where $s \in \mathcal{S}, a \in \mathcal{A}$ and $r_1, \ldots, r_n, r \in \mathbb{Q}$. $[\phi_1]\phi_2$ is interpreted as "ϕ_2 is the case after everyone simultaneously and commonly learns that ϕ_1 is the case." Let $\Phi_{\mathcal{L}_{\mathsf{PUPDEL}}}$ denote the set of all well-formed formulae of $\mathcal{L}_{\mathsf{PUPDEL}}$. Let $P_{a,\mathcal{L}_{\mathsf{PUPDEL}}}$ denote the set of all a-probability formulae of $\mathcal{L}_{\mathsf{PUPDEL}}$ and let $T_{\mathcal{L}_{\mathsf{PUPDEL}}}$ denote the set of all terms of $\mathcal{L}_{\mathsf{PUPDEL}}$.

3.2 Semantics

Based on Definition 2 and [[7]: 388] , we define an updated multi-agent structured Kripke model for PUPDEL as follows:

[8] [[1]: 357–359].
[9] [[7]: 387].

Definition 7. *When a multi-agent structured Kripke model* $\mathcal{M}_{\mathsf{PUPDEL}} := (W, \pi, R_{a_1}, \ldots, R_{a_n}, \mathcal{P})$ *and* $\phi_1 \in \Phi_{\mathcal{L}_{\mathsf{PUPDEL}}}$ *are given, an updated multi-agent structured Kripke model for* PUPDEL $\mathcal{M}_{\phi_1,\mathsf{PUPDEL}}$ *is an* $(n+3)$*-tuple* $(W_{\phi_1}, \pi_{\phi_1}, R_{a_1,\phi_1}, \ldots, R_{a_n,\phi_1}, \mathcal{P}_{\phi_1})$*, where*

$$
\begin{aligned}
& W_{\phi_1} = W, \\
& \pi_{\phi_1} = \pi, \\
& R_{a_1,\phi_1} = \{(w_1, w_2) \in R_{a_1} : (\mathcal{M}_{\mathsf{PUPDEL}}, w_2) \models \phi_1\}, \\
& \quad \vdots \\
& R_{a_n,\phi_1} = \{(w_1, w_2) \in R_{a_n} : (\mathcal{M}_{\mathsf{PUPDEL}}, w_2) \models \phi_1\}, \\
& \mathcal{P}_{\phi_1} := (W_{a,w_1,\phi_1}, \mathcal{F}_{a,w_1,\phi_1}, P_{a,w_1,\phi_1}), \textit{ where} \\
& \quad W_{a,w_1,\phi_1} := \begin{cases} W_{a,w_1} & \textit{if } P_{a,w_1}(W_{a,w_1}(\phi_1)) = 0, \\ \{w_2 \in W_{a,w_1} : (\mathcal{M}_{\mathsf{PUPDEL}}, w_2) \models \phi_1\} & \textit{otherwise,} \end{cases} \\
& \quad \mathcal{F}_{a,w_1,\phi_1} \textit{ is a } \sigma\textit{-field of subsets of } W_{a,w_1,\phi_1}, \\
& \quad P_{a,w_1,\phi_1}(W_{a,w_1,\phi_1}(\phi_2)) := \begin{cases} P_{a,w_1}(W_{a,w_1}(\phi_2)) & \textit{if } P_{a,w_1}(W_{a,w_1}(\phi_1)) = 0, \\ \dfrac{P_{a,w_1}(W_{a,w_1}(\phi_1 \wedge \phi_2))}{P_{a,w_1}(W_{a,w_1}(\phi_1))} & \textit{otherwise,} \end{cases}
\end{aligned}
$$

Moreover, $\mathcal{M}_{\mathsf{PUPDEL}}$ *and* $\mathcal{M}_{\phi_1,\mathsf{PUPDEL}}$ *satisfies (CONS),(OBJ),(SDP),(UNIF) and (MEAS). Let* $\mathbb{M}_{\mathsf{PUPDEL}}$ *denote the class of all structured Kripke models for* PUPDEL*.*

Based on [[7]: 388], we give the following truth definition.

Definition 8. *The notion of* $\phi \in \Phi_{\mathcal{L}_{\mathsf{PUPDEL}}}$ *being true at* $w \in W$ *in* $\mathcal{M}_{\mathsf{PUPDEL}}$*, in symbols* $(\mathcal{M}_{\mathsf{PUPDEL}}, w) \models \phi$ *is inductively defined as follows:*

$$
\begin{aligned}
& (\mathcal{M}_{\mathsf{PUPDEL}}, w) \models s \quad \textit{iff} \quad \pi(w)(s) = \mathbf{true}, \\
& (\mathcal{M}_{\mathsf{PUPDEL}}, w) \models \phi_1 \wedge \phi_2 \quad \textit{iff} \quad (\mathcal{M}_{\mathsf{PUPDEL}}, w) \models \phi_1 \quad \textit{and} \quad (\mathcal{M}_{\mathsf{PUPDEL}}, w) \models \phi_2, \\
& (\mathcal{M}_{\mathsf{PUPDEL}}, w) \models \neg\phi \quad \textit{iff} \quad (\mathcal{M}_{\mathsf{PUPDEL}}, w) \not\models \phi, \\
& (\mathcal{M}_{\mathsf{PUPDEL}}, w_1) \models K_a(\phi) \quad \textit{iff} \quad (\mathcal{M}_{\mathsf{PUPDEL}}, w_2) \models \phi \quad \textit{for all } w_2 \in R_a(w_1), \\
& (\mathcal{M}_{\mathsf{PUPDEL}}, w) \models \sum_{i=1}^{n} r_i \mathbf{P}_a(\phi_i) \geq r \quad \textit{iff} \quad \sum_{i=1}^{n} r_i P_{a,w}(W_{a,w}(\phi_i)) \geq r, \\
& (\mathcal{M}_{\mathsf{PUPDEL}}, w) \models [\phi_1]\phi_2 \quad \textit{iff} \quad (\mathcal{M}_{\phi_1,\mathsf{PUPDEL}}, w) \models \phi_2.
\end{aligned}
$$

If $(\mathcal{M}_{\mathsf{PUPDEL}}, w) \models \phi$ *for all* $w \in W$*, we write* $\mathcal{M}_{\mathsf{PUPDEL}} \models \phi$ *and say that* ϕ *is valid in* $\mathcal{M}$*. If* ϕ *is valid in all models in* $\mathbb{M}_{\mathsf{PUPDEL}}$*, we write* $\mathbb{M}_{\mathsf{PUPDEL}} \models \phi$ *and say that* ϕ *is valid with respect to* $\mathbb{M}_{\mathsf{PUPDEL}}$*.*

3.3 Syntax

Besides (A1),(A2),(A3),(A4),(A5),(A6),(A7),(A8),(A9),(A10),(A11),(A12),(A13), (A14),(A15),(A16),(A17) and (A18), the axiom system of PUPDEL has the following axioms based on [[7]: 395]:

$$\begin{array}{ll}
\text{(A19)} & [\phi_1](\phi_2 \rightarrow \phi_3) \rightarrow ([\phi_1](\phi_2) \rightarrow [\phi_1](\phi_3)) \quad \text{(K)}, \\
\text{(A20)} & \neg[\phi_1]\phi_2 \leftrightarrow [\phi_1]\neg\phi_2 \quad \text{(Functionality)}, \\
\text{(A21)} & s \leftrightarrow [\phi]s \quad \text{(Atomic Permanence)}, \\
\text{(A22)} & [\phi_1]\mathbf{K}_a(\phi_2) \leftrightarrow \mathbf{K}_a(\phi_1 \rightarrow [\phi_1]\phi_2) \quad \text{(Knowledge Update)}, \\
 & \mathbf{P}_a(\phi) > 0 \rightarrow \\
\text{(A23)} & (([\phi]\sum_{i=1}^{n} r_i\mathbf{P}_a(\phi_i) \geq r) \leftrightarrow (\sum_{i=1}^{n} r_i\mathbf{P}_a(\phi \wedge [\phi]\phi_i) \geq r\mathbf{P}_a(\phi))) \\
 & \text{(Probability Update 1)}, \\
 & \mathbf{P}_a(\phi) = 0 \rightarrow \\
\text{(A24)} & (([\phi]\sum_{i=1}^{n} r_i\mathbf{P}_a(\phi_i) \geq r) \leftrightarrow (\sum_{i=1}^{n} r_i\mathbf{P}_a([\phi]\phi_i) \geq r)) \\
 & \text{(Probability Update 2)}.
\end{array}$$

Besides (R1),(R2) and (R3), the axiom system of PUPDEL has the following inference rule based on [[7]: 395]:

$$\text{(R4)} \quad \frac{\phi_2}{[\phi_1]\phi_2} \quad \text{(Generalisation)}.$$

If $\phi \in \mathit{\Phi}_{\mathsf{PUPDEL}}$ is provable by (R1),(R2),(R3) or (R4) from (A1),(A2),(A3),(A4),(A5),(A6),(A7),(A8),(A9),(A10),(A11),(A12),(A13),(A14),(A15),(A16),(A17),(A18),(A19),(A20),(A21),(A22),(A23) or (A24), we write $\vdash_{\mathsf{PUPDEL}} \phi$.

3.4 Monty Hall Dilemma

The *Monty Hall dilemma* is an open problem which is well-known among linguists, philosophers, psychologists, and logicians. Nowadays this dilemma is a common topic in probabilistic dynamic epistemic logic. Kooi gave an answer to this dilemma in terms of PUPDEL. This dilemma is stated as follows:[10]

Example 1. Suppose you're on a game show, and you're given the choice of three doors. Behind the door is a car, behind the others, goats. You pick a door, say number 1, and the host (Monty Hall), who knows what's behind the door, opens another door, say number 3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors?

3.5 Semantic Analysis of Monty Hall Dilemma in Terms of PUPDEL

Assume that $\mathcal{M}_{\mathsf{PUPDEL}} := (W, \pi, R_I, R_{MH}, \mathcal{P})$ is given. Let W be $\{w_1, w_2, w_3, w_4\}$ where w_1 is a world where there is a car behind the door 1 and MH opens the door 2, w_2 is a world where there is a car behind the door 1 and MH opens the door 3, w_3 is a world where there is a car behind the door 2 and MH opens the door 3, and w_4 is a world where there is a car behind the door 3 and MH opens the door 2.

[10] [[14]: 6].

Because $w_1 \in R_I(w_2)$ for all $w_1, w_2 \in W$, from (SDP) we have

$$\mathcal{P}(I, w_1) = \mathcal{P}(I, w_2) = \mathcal{P}(I, w_3) = \mathcal{P}(I, w_4).$$

Then we have, for all $w \in W$,

$$P_{I,w}(\{w_1\}) = P_{I,w}(\{w_2\}) = \frac{1}{6}, \quad P_{I,w}(\{w_3\}) = P_{I,w}(\{w_4\}) = \frac{1}{3}.$$

Let ϕ_3, ψ_1 and ψ_2 each denote the following sentence.

ϕ_3 := MH opens the door 3,
ψ_1 := there is a car behind the door 1,
ψ_2 := there is a car behind the door 2.

Because

$$P_{I,w}(W_{I,w}(\psi_1)) = P_{I,w}(\{w_1\}) + P_{I,w}(\{w_2\}) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3},$$

we have, for all $w \in W$,

$$(\mathcal{M}_{\mathsf{PUPDEL}}, w') \models \mathbf{P}_I(\psi_1) = \tfrac{1}{3} \quad \text{for all } w' \in R_I(w).$$

So we have

$$\mathcal{M}_{\mathsf{PUPDEL}} \models \mathbf{K}_I(\mathbf{P}_I(\psi_1) = \frac{1}{3}).$$

Moreover, because, for all $w \in W$,

$$P_{I,w,\phi_3}(W_{I,w,\phi_3}(\psi_1)) = \frac{P_{I,w}(W_{I,w}(\phi_3 \wedge \psi_1))}{P_{I,w}(W_{I,w}(\phi_3))} = \frac{P_{I,w}(\{w_2\})}{P_{I,w}(\{w_2\}) + P_{I,w}(\{w_3\})} = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{3}} = \frac{1}{3},$$
$$P_{I,w,\phi_3}(W_{I,w,\phi_3}(\psi_2)) = \frac{P_{I,w}(W_{I,w}(\phi_3 \wedge \psi_2))}{P_{I,w}(W_{I,w}(\phi_3))} = \frac{P_{I,w}(\{w_3\})}{P_{I,w}(\{w_2\}) + P_{I,w}(\{w_3\})} = \frac{\frac{1}{3}}{\frac{1}{6}+\frac{1}{3}} = \frac{2}{3},$$

we have, for all $w \in W$,

$$(\mathcal{M}_{\phi_3,\mathsf{PUPDEL}}, w') \models \mathbf{P}_I(\psi_1) = \tfrac{1}{3} \quad \text{for all } w' \in R_{I,\phi_3}(w),$$
$$(\mathcal{M}_{\phi_3,\mathsf{PUPDEL}}, w') \models \mathbf{P}_I(\psi_2) = \tfrac{2}{3} \quad \text{for all } w' \in R_{I,\phi_3}(w).$$

So we have the following results:

$$\mathcal{M}_{\mathsf{PUPDEL}} \models [\phi_3]\mathbf{K}_I(\mathbf{P}_I(\psi_1) = \frac{1}{3}), \quad \mathcal{M}_{\mathsf{PUPDEL}} \models [\phi_3]\mathbf{K}_I(\mathbf{P}_I(\psi_2) = \frac{2}{3}).$$

Therefore I should switch my choice. However, Ichikawa comments that there are overwhelmingly many examinees that support the answer that they do not have to switch their choices.[11] Isn't there a method of representing the belief change that supports this answer? There exists such a method. General imaging is a prime candidate for this task. I will propose a new version of probabilistic dynamic epistemic logic (GIPDEL) that is based on general imaging.

[11] [[5]: 27].

4 General-Imaging-Based Probabilistic Dynamic Epistemic Logic **GIPDEL**

4.1 Language

I give the language of GIPDEL $\mathcal{L}_{\mathsf{GIPDEL}}$.

Definition 9. *$\mathcal{L}_{\mathsf{GIPDEL}}$ is defined in terms of a finite set S of sentential variables, a finite set A of agents, an epistemic operator $\mathbf{K}_a$, a characteristic function $\mathbf{G}_i$, a probability function symbol $\mathbf{P}_a$ and an update operator $[\ ]$. The well-formed formulae of $\mathcal{L}_{\mathsf{GIPDEL}}$ are given by the following rule:*

$$\phi ::= s \mid \top \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \mathbf{K}_a(\phi) \mid \sum_{i=1}^{n} r_i \mathbf{P}_a(\phi_i) \geq r \mid \sum_{i=1}^{n} r_i \mathbf{G}_i(\phi) = r \mid [\phi_1]\phi_2,$$

where $s \in \mathcal{S}, a \in \mathcal{A}$ and $r_1, \ldots, r_n, r \in \mathbb{Q}$. Let $\Phi_{\mathcal{L}_{\mathsf{GIPDEL}}}$ denote the set of all well-formed formulae of $\mathcal{L}_{\mathsf{GIPDEL}}$. Let $P_{a,\mathcal{L}_{\mathsf{GIPDEL}}}$ denote the set of all a-probability formulae of $\mathcal{L}_{\mathsf{GIPDEL}}$ and let $T_{\mathcal{L}_{\mathsf{GIPDEL}}}$ denote the set of all terms of $\mathcal{L}_{\mathsf{GIPDEL}}$.

4.2 Semantics

We prepare some concepts for the definition of general imaging. Let $w_2 \preceq_{w_1} w_3$ denote that $w_2 \in W$ is at least similar to $w_1 \in W$ as $w_3 \in W$ is. Let $w_2 \prec_{w_1} w_3$ denote that $w_2 \in W$ is more similar to $w_1 \in W$ than $w_3 \in W$ is. The comparative similarity system is defined as follows:[12]

Definition 10. *We posit an assignment of $\preceq_w$ and $R_a(w)$ to $w \in W$. Let us call such an assignment a comparative similarity system iff, for each $w_1, w_2, w_3, w_4 \in W$, the following six conditions hold.*

1. *$\preceq_{w_1}$ is transitive; that is, if $w_2 \preceq_{w_1} w_3$ and $w_3 \preceq_{w_1} w_4$, then $w_2 \preceq_{w_1} w_4$.*
2. *$\preceq_{w_1}$ is strongly connected; that is, for any w_2 and w_3, $w_2 \preceq_{w_1} w_3$ or $w_3 \preceq_{w_1} w_2$.*
3. *w_1 is self-accessible; that is, $w_1 \in R_a(w_1)$.*
4. *w_1 is strictly $\preceq_{w_1}$-minimal; that is, if any w_2 different from w_1, $w_1 \prec_{w_1} w_2$.*
5. *Inaccessible worlds are strictly $\preceq_{w_1}$-maximal; that is, if $w_3 \notin R_a(w_1)$, then for any w_2, $w_2 \preceq_{w_1} w_3$.*
6. *Accessible worlds are more similar to w_1 than inaccessible worlds; if $w_2 \in R_a(w_1)$ and $w_3 \notin R_a(w_1)$, $w_2 \prec_{w_1} w_3$.*

Imaging is a method of changing probability functions Lewis proposed in [9]. It produces minimal disturbance in the following sense:

> Imaging P on A gives a minimal revision in this sense: unlike all other revisions of P to make A certain, it involves no gratuitous movement of probability from worlds to dissimilar worlds.[13]

[12] This definition is based on [[8]: 48–49].

[13] [[9]: 148].

In order to define imaging, it is necessary to assume that, in a comparative similarity system, for any $w \in W$, there is a unique $w' \in W$ that is the most similar to w among the worlds where ϕ is true. General imaging is a version of imaging Gärdenfors proposed in [3]. In order to define general imaging, we have only to assume that, in a comparative similarity system, for any $w \in W$, there is at least one world that is the most similar to w among the worlds where ϕ is true. Let W_ϕ^w denote the set of all worlds that are the most similar to w among the worlds where ϕ is true. We define $g_{W_\phi^{w_1}} : W \longrightarrow \mathbb{R}$ as follows:

Definition 11.

$$g_{W_\phi^{w_1}}(w_2) := \begin{cases} \dfrac{1}{|W_\phi^{w_1}|} & \textit{if } w_2 \in W_\phi^{w_1}, \\ 0 & \textit{otherwise}, \end{cases} \quad \textit{where} \sum_{w_2 \in W_\phi^{w_1}} g_{W_\phi^{w_1}}(w_2) = 1.$$

By means of Definition 4 and Definition 11, we define general imaging as follows:
Definition 12.

$$P^{\odot}_{a,w_1,\phi_1}(W_{a,w_1,\phi_1}(\phi_2))$$
$$:= \begin{cases} \displaystyle\sum_{w_3 \in W_{a,w_1}(\phi_2)} \sum_{w_2 \in W_{a,w_1}} \left(g_{W^{w_2}_{a,w_1,\phi_1}}(w_3) \cdot P_{a,w_1}(\{w_2\}) \right) & \textit{if } \vdash_{\mathsf{GIPDEL}} \phi_1 \not\leftrightarrow \bot \not\leftrightarrow \phi_2, \\ 0 & \textit{if } \vdash_{\mathsf{GIPDEL}} \phi_1 \not\leftrightarrow \bot \leftrightarrow \phi_2, \\ 1 & \textit{if } \vdash_{\mathsf{GIPDEL}} \phi_1 \leftrightarrow \bot. \end{cases}$$

We define an updated multi-agent structured Kripke model for GIPDEL as follows:

Definition 13. *When a multi-agent structured Kripke model $\mathcal{M}_{\mathsf{GIPDEL}} := (W, \preceq, \pi, R_{a_1}, \ldots, R_{a_n}, \tilde{\mathcal{P}})$, where W is a finite set of possible worlds and $\tilde{\mathcal{P}}$ is an extended probability assignment that assigns to each $a \in A$ and each $w \in W$ an extended probability space $\tilde{\mathcal{P}}(a, w) = (W_{a,w}, \mathcal{F}_{a,w}, 1, 0, P_{a,w})$, where $\mathcal{F}_{a,w}$ is a field of subsets of $W_{a,w}$, and $\phi_1 \in \Phi_{\mathcal{L}_{\mathsf{GIPDEL}}}$ are given, an updated multi-agent structured Kripke model for GIPDEL $\mathcal{M}_{\phi_1,\mathsf{GIPDEL}}$ is an $(n+4)$-tuple $(W_{\phi_1}, \preceq_{\phi_1}, \pi_{\phi_1}, R_{a_1,\phi_1}, \ldots, R_{a_n,\phi_1}, \tilde{\mathcal{P}}_{\phi_1})$, where*

$W_{\phi_1} = W$,
$\preceq_{\phi_1} = \preceq$ *(defined by Definition 10)*,
$\pi_{\phi_1} = \pi$,
$R_{a_1,\phi_1} = \{(w_1, w_2) \in R_{a_1} : (\mathcal{M}_{\mathsf{GIPDEL}}, w_2) \models \phi_1\}$,
$\vdots$
$R_{a_n,\phi_1} = \{(w_1, w_2) \in R_{a_n} : (\mathcal{M}_{\mathsf{GIPDEL}}, w_2) \models \phi_1\}$,
$\tilde{\mathcal{P}}_{\phi_1} := (W_{a,w_1,\phi_1}, \mathcal{F}_{a,w_1,\phi_1}, g_{W^{w_2}_{a,w_1,\phi_1}}, P_{a,w_1,\phi_1})$, *where*

$$W_{a,w_1,\phi_1} := \begin{cases} W_{a,w_1} & \textit{if } P_{a,w_1}(W_{a,w_1}(\phi_1)) = 0, \\ \{w_2 \in W_{a,w_1} : (\mathcal{M}_{\mathsf{GIPDEL}}, w_2) \models \phi_1\} & \textit{otherwise}, \end{cases}$$

$\mathcal{F}_{a,w_1,\phi_1}$ *is a field of subsets of* W_{a,w_1,ϕ_1},
$g_{W^{w_2}_{a,w_1,\phi_1}}$ *was defined by Definition 11,*

$$P_{a,w_1,\phi_1}(W_{a,w_1,\phi_1}(\phi_2)) := \begin{cases} P_{a,w_1}(W_{a,w_1}(\phi_2)) & \textit{if } P_{a,w_1}(W_{a,w_1}(\phi_1)) = 0, \\ P^{\odot}_{a,w_1,\phi_1}(W_{a,w_1,\phi_1}(\phi_2)) \ \textit{(defined by Definition 12)} & \textit{otherwise}, \end{cases}$$

Moreover, $\mathcal{M}_{\mathsf{GIPDEL}}$ and $\mathcal{M}_{\phi_1,\mathsf{GIPDEL}}$ satisfies (CONS),(OBJ),(SDP),(UNIF) and (MEAS). Let $\mathbb{M}_{\mathsf{GIPDEL}}$ denote the class of all structured Kripke models for GIPDEL.

I give the following truth definition.

Definition 14. *The notion of $\phi \in \Phi_{\mathcal{L}_{\mathsf{GIPDEL}}}$ being true at $w \in W$ in $\mathcal{M}_{\mathsf{GIPDEL}}$, in symbols $(\mathcal{M}_{\mathsf{GIPDEL}}, w) \models \phi$ is inductively defined as follows:*

$$(\mathcal{M}_{\mathsf{GIPDEL}}, w) \models s \quad \textit{iff} \quad \pi(w)(s) = \mathbf{true},$$
$$(\mathcal{M}_{\mathsf{GIPDEL}}, w) \models \phi_1 \wedge \phi_2 \quad \textit{iff} \quad (\mathcal{M}_{\mathsf{GIPDEL}}, w) \models \phi_1 \quad \textit{and} \quad (\mathcal{M}_{\mathsf{GIPDEL}}, w) \models \phi_2,$$
$$(\mathcal{M}_{\mathsf{GIPDEL}}, w) \models \neg\phi \quad \textit{iff} \quad (\mathcal{M}_{\mathsf{GIPDEL}}, w) \not\models \phi,$$
$$(\mathcal{M}_{\mathsf{GIPDEL}}, w_1) \models K_a(\phi) \quad \textit{iff} \quad (\mathcal{M}_{\mathsf{GIPDEL}}, w_2) \models \phi \quad \textit{for all } w_2 \in R_a(w_1),$$
$$(\mathcal{M}_{\mathsf{GIPDEL}}, w) \models \sum_{i=1}^{2^n} r_i \mathbf{G}_i(\phi) = r \quad \textit{iff} \quad \sum_{i=1}^{2^n} r_i \cdot \left\{ \begin{matrix} 1 \textit{ if } (\mathcal{M}_{\mathsf{GIPDEL}}, w_i) \models \phi \\ 0 \quad \textit{otherwise} \end{matrix} \right\} = r,$$
$(\textit{where } S := \{s_1, \ldots, s_n\} \textit{ and } (\mathcal{M}_{\mathsf{GIPDEL}}, w_1) \models s_1 \& \ldots \& s_n \textit{ and } (\mathcal{M}_{\mathsf{GIPDEL}}, w_2) \models \neg s_1 \& \ldots \& s_n \textit{ and } \ldots \textit{ and } (\mathcal{M}_{\mathsf{GIPDEL}}, w_{2^n-1}) \models \neg s_1 \& \ldots \neg s_{n-1} \& s_n \textit{ and } (\mathcal{M}_{\mathsf{GIPDEL}}, w_{2^n}) \models \neg s_1 \& \ldots \& \neg s_{n-1} \& \neg s_n,)$
$$(\mathcal{M}_{\mathsf{GIPDEL}}, w) \models \sum_{i=1}^{n} r_i \mathbf{P}_a(\phi_i) \geq r \quad \textit{iff} \quad \sum_{i=1}^{n} r_i P_{a,w}(W_{a,w}(\phi_i)) \geq r,$$
$$(\mathcal{M}_{\mathsf{GIPDEL}}, w) \models [\phi_1]\phi_2 \quad \textit{iff} \quad (\mathcal{M}_{\phi_1,\mathsf{GIPDEL}}, w) \models \phi_2.$$

If $(\mathcal{M}_{\mathsf{GIPDEL}}, w) \models \phi$ for all $w \in W$, we write $\mathcal{M}_{\mathsf{GIPDEL}} \models \phi$ and say that ϕ is valid in $\mathcal{M}$. If ϕ is valid in all models in $\mathbb{M}_{\mathsf{GIPDEL}}$, we write $\mathbb{M}_{\mathsf{GIPDEL}} \models \phi$ and say that ϕ is valid with respect to $\mathbb{M}_{\mathsf{GIPDEL}}$.

4.3 Syntax

The axiom system of GIPDEL is the same as that of PUPDEL, except that the former has the following axioms and inference rule, instead of (A23) and (A24).

(A25) $$(\sum_{i=1}^{n} r_i \mathbf{G}_i(\phi) = r) \leftrightarrow (\sum_{i=1}^{n} r_i \mathbf{G}_i(\phi) + 0\mathbf{G}_{n+1}(\phi) = r)$$
(Adding and Deleting 0 Terms),

(A26) $$(\sum_{i=1}^{n} r_i \mathbf{G}_i(\phi) = r) \rightarrow (\sum_{i=1}^{n} r_{j_i} \mathbf{G}_{j_i}(\phi) = r)$$
if $j_1, \ldots, j_n$ is a permutation of $1, \ldots, n$ (Permutation),

(A27) $$(\sum_{i=1}^{n} r_i \mathbf{G}_i(\phi) = r) \wedge (\sum_{i=1}^{n} r'_i \mathbf{G}_i(\phi) = r') \rightarrow \sum_{i=1}^{n} (r_i + r'_i) \mathbf{G}_i(\phi) = (r + r')$$
(Addition of Coefficients),

(A28) $$(\sum_{i=1}^{n} r_i \mathbf{G}_i(\phi) = r) \leftrightarrow (\sum_{i=1}^{n} r' r_i \mathbf{G}_i(\phi) = r' r)$$
if $r' > 0$ (Multiplication of Nonzero Coefficients),

(A29) $$([\phi_1] \sum_{i=1}^{n} r_i \mathbf{G}_i(\phi_2) = r) \leftrightarrow (\sum_{i=1}^{n} r_i \mathbf{G}_i([\phi_1]\phi_2) = r)$$
(Characteristic Function Update),

(A30) $$([\phi] \sum_{i=1}^{n} r_i \mathbf{P}_a(\phi_i) \geq r) \leftrightarrow (\sum_{i=1}^{n} r_i \mathbf{P}_a([\phi]\phi_i) \geq r)$$
(Probability Update),

$$\text{(A31)}\quad \begin{array}{l}(\mathbf{P}_a(s_1\&\ldots\&s_n)=r_1\&\mathbf{P}_a(\neg s_1\&\ldots\&s_n)=r_2\&\ldots\\ \&\mathbf{P}_a(\neg s_1\&\ldots\&\neg s_{n-1}\&s_n)=r_{2^n-1}\&\mathbf{P}_a(\neg s_1\&\ldots\&\neg s_{n-1}\&\neg s_n)=r_{2^n}\\ \&\mathbf{P}_a(\phi_1)=\sum_{i=1}^{2^n} r_i\mathbf{G}_i(\phi_1))\rightarrow([\phi_2]\mathbf{P}_a(\phi_1)=\sum_{i=1}^{2^n} r_i\mathbf{G}_i(\phi_1)),\\ \text{where } S:=\{s_1,\ldots,s_n\}\quad \text{(Linearity of Probability Update),}\end{array}$$

$$\text{(R5)}\quad \frac{\phi_1\leftrightarrow\phi_2}{\mathbf{G}_i(\phi_1)=\mathbf{G}_i(\phi_2)}\quad \text{(Distributivity).}$$

If $\phi\in\Phi_{\mathsf{GIPDEL}}$ is provable by (R1),(R2),(R3),(R4) or (R5) from (A1),(A2),(A3),(A4),(A5),(A6),(A7),(A8),(A9),(A10),(A11),(A12),(A13),(A14),(A15),(A16),(A17),(A18),(A19),(A20),(A21),(A22),(A25),(A26),(A27),(A28),(A29),(A30) or (A31), we write $\vdash_{\mathsf{GIPDEL}}\phi$.

4.4 Soundness and Completeness

We can prove the soundness theorem of GIPDEL in the usual way.

Theorem 3. ***(Soundness)***
If $\vdash_{\mathsf{GIPDEL}}\phi$*, then* $\mathbb{M}_{\mathsf{GIPDEL}}\models\phi$*.*

In order to prove completeness of GIPDEL, we give a translation function $\tau:\mathcal{L}_{\mathsf{GIPDEL}}\rightarrow\mathcal{L}_{\mathsf{PEL}}$. Because completeness of PEL is proved, it suffices to show that every well-formed formula is equivalent to its translation in GIPDEL. This method is usual in the literature of dynamic epistemic logics.[14]

Definition 15. *A translation function* $\tau:\mathcal{L}_{\mathsf{GIPDEL}}\rightarrow\mathcal{L}_{\mathsf{PEL}}$ *is defined as follows:*

1. $\tau(s)=s$,
2. $\tau(\top)=\top$,
3. $\tau(\neg\phi)=\neg\tau(\phi)$,
4. $\tau(\phi_1\wedge\phi_1)=\tau(\phi_1)\wedge\tau(\phi_2)$,
5. $\tau(\mathbf{K}_a(\phi))=\mathbf{K}_a(\tau(\phi))$,
6. $\tau(\sum_{i=1}^{n} r_i\mathbf{G}_i(\phi)=r)=(\sum_{i=1}^{n} r_i\mathbf{G}_i(\tau(\phi))=r)$,
7. $\tau(\sum_{i=1}^{n} r_i\mathbf{P}_a(\phi_i)\geq r)=(\sum_{i=1}^{n} r_i\mathbf{P}_a(\tau(\phi_i))\geq r)$,
8. $\tau([\phi]s)=s$,
9. $\tau([\phi_1]\neg\phi_2)=\neg\tau([\phi_1]\phi_2)$,
10. $\tau([\phi_1](\phi_2\wedge\phi_2))=\tau([\phi_1]\phi_2)\wedge\tau([\phi_1]\phi_3)$,
11. $\tau([\phi_1]\mathbf{K}_a(\phi_2))=\mathbf{K}_a(\tau(\phi_1)\rightarrow\tau([\phi_1]\phi_2))$,
12. $\tau([\phi_1]\sum_{i=1}^{n} r_i\mathbf{G}_i(\phi_2)=r)=(\sum_{i=1}^{n} r_i\mathbf{G}_i(\tau([\phi_1]\phi_2))=r)$,
13. $\tau([\phi]\sum_{i=1}^{n} r_i\mathbf{P}_a(\phi_i)\geq r)=(\sum_{i=1}^{n} r_i\mathbf{P}_a(\tau([\phi]\phi_i))\geq r)$.

We can prove the following lemma.

[14] As for this method, refer to [[4]: 95-97], [[6]: 110-113] and [[7]: 396-397].

Lemma 1. *For every* $\phi \in \Phi_{\mathsf{GIPDEL}}$, $\vdash_{\mathsf{GIPDEL}} \tau(\phi) \leftrightarrow \phi$.

From Theorem 2 and Lemma 1, we can prove the completeness theorem of GIPDEL.

Theorem 4. ***(Completeness)***
If $\mathbb{M}_{\mathsf{GIPDEL}} \models \phi$, *then* $\vdash_{\mathsf{GIPDEL}} \phi$.

4.5 Semantic Analysis of Monty Hall Dilemma in Terms of GIPDEL

Assume that $\mathcal{M}_{\mathsf{GIPDEL}} := (W, \preceq, \pi, R_I, R_{MH}, \tilde{\mathcal{P}})$ is given. Then $\preceq$ enables us to assume that w_2 is the most similar to w_1 among the worlds where ϕ_3 is true, and to assume that w_2 and w_3 are the most similar to w_4 among the worlds where ϕ_3 is true.

Because, for all $w \in W$,

$$P_{I,w,\phi_3}(W_{I,w,\phi_3}(\psi_1)) = P^{\odot}_{I,w,\phi_3}(W_{I,w,\phi_3}(\psi_1)) = \tfrac{1}{6} + \tfrac{1}{6} + \tfrac{1}{2} \cdot \tfrac{1}{3} = \tfrac{1}{2},$$
$$P_{I,w,\phi_3}(W_{I,w,\phi_3}(\psi_2)) = P^{\odot}_{I,w,\phi_3}(W_{I,w,\phi_3}(\psi_2)) = \tfrac{1}{3} + \tfrac{1}{2} \cdot \tfrac{1}{3} = \tfrac{1}{2},$$

we have, for all $w \in W$,

$$(\mathcal{M}_{\phi_3,\mathsf{GIPDEL}}, w') \models \mathbf{P}_I(\psi_1) = \tfrac{1}{2} \quad \text{for all } w' \in R_{I,\phi_3}(w),$$
$$(\mathcal{M}_{\phi_3,\mathsf{GIPDEL}}, w') \models \mathbf{P}_I(\psi_2) = \tfrac{1}{2} \quad \text{for all } w' \in R_{I,\phi_3}(w).$$

So we have the following results:

$$\mathcal{M}_{\mathsf{GIPDEL}} \models [\phi_3]\mathbf{K}_I(\mathbf{P}_I(\psi_1) = \frac{1}{2}), \quad \mathcal{M}_{\mathsf{GIPDEL}} \models [\phi_3]\mathbf{K}_I(\mathbf{P}_I(\psi_2) = \frac{1}{2}).$$

Therefore I do not have to switch my choice. In this way, general imaging can represent the belief change that supports this answer.

5 Semantic Analyses of Modified Version of Monty Hall Dilemma

5.1 Modified Version of Monty Hall Dilemma

Ichikawa presented a modified version of the problem of three prisoners.[15] Likewise, we can state a modified version of the Monty Hall dilemma as follows:

Example 2. Suppose you're on a game show, and you're given the choice of three doors. Behind the door is a car, behind the others, goats. Somehow, you know the probability of there being a car behind the door 1 is $\frac{1}{4}$, the probability of there being one behind the door 2 is $\frac{1}{4}$ and the probability of there being one behind the door 3 is $\frac{1}{2}$. You only tentatively pick a door, say number 1, and the host (Monty Hall), who knows what's behind the door, opens another door, say number 2, which has a goat. Then what do you think the probability of there being a car behind the door 1?

[15] [[5]: 29].

5.2 Semantic Analysis in Terms of PUPDEL

Assume that $\mathcal{M}_{\mathsf{PUPDEL}} := (W, \pi, R_I, R_{MH}, \mathcal{P})$ is given. Then we have, for all $w \in W$,

$$P_{I,w}(\{w_1\}) = P_{I,w}(\{w_2\}) = \frac{1}{8}, \quad P_{I,w}(\{w_3\}) = \frac{1}{4}, \quad P_{I,w}(\{w_4\}) = \frac{1}{2}.$$

Let ϕ_2 denote the sentence that MH opens the door 2.

Because, for all $w \in W$,

$$P_{I,w,\phi_2}(W_{I,w,\phi_2}(\psi_1)) = \frac{P_{I,w}(W_{I,w}(\phi_2 \wedge \psi_1))}{P_{I,w}(W_{I,w}(\phi_2))} = \frac{P_{I,w}(\{w_1\})}{P_{I,w}(\{w_1\}) + P_{I,w}(\{w_4\})} = \frac{\frac{1}{8}}{\frac{1}{8} + \frac{1}{2}} = \frac{1}{5},$$

we have, for all $w \in W$,

$$(\mathcal{M}_{\phi_2,\mathsf{PUPDEL}}, w') \models \mathbf{P}_I(\psi_1) = \tfrac{1}{5} \quad \text{for all } w' \in R_{I,\phi_2}(w).$$

So we have the following result:

$$\mathcal{M}_{\mathsf{PUPDEL}} \models [\phi_2]\mathbf{K}_I(\mathbf{P}_I(\psi_1) = \frac{1}{5}).$$

However, Ichikawa comments that there is no reason to believe that the probability that there is a car behind the door 1 decreases even though there may be reason to believe that the probability remains.[16]

5.3 Semantic Analysis in Terms of GIPDEL

Assume that $\mathcal{M}_{\mathsf{GIPDEL}} := (W, \preceq, \pi, R_I, R_{MH}, \tilde{\mathcal{P}})$ is given. Then $\preceq$ enables us to assume that w_1 is the most similar to w_2 among the worlds where ϕ_2 is true, and to assume that w_1 and w_4 are the most similar to w_3 among the worlds where ϕ_2 is true.

Because, for all $w \in W$,

$$P_{I,w,\phi_2}(W_{I,w,\phi_2}(\psi_1)) = P^{\odot}_{I,w,\phi_2}(W_{I,w,\phi_2}(\psi_1)) = \frac{1}{8} + \frac{1}{8} + \frac{1}{2} \cdot \frac{1}{4} = \frac{3}{8},$$

we have, for all $w \in W$,

$$(\mathcal{M}_{\phi_2,\mathsf{GIPDEL}}, w') \models \mathbf{P}_I(\psi_1) = \tfrac{3}{8} \quad \text{for all } w' \in R_{I,\phi_2}(w).$$

So we have the following result:

$$\mathcal{M}_{\mathsf{GIPDEL}} \models [\phi_2]\mathbf{K}_I(\mathbf{P}_I(\psi_1) = \frac{3}{8}).$$

GIPDEL can give a plausible answer also in Example 2.

[16] [[5]: 30].

6 Concluding Remarks

Kooi says:

> The Monty Hall dilemma is a puzzle for which intuitions fail many people. The best way to show that the counterintuitive results are correct is to use some formal method. PDEL provides such a method.[17]

I do not agree with him. The Monty Hall dilemma does not illustrate that intuitive reasoning is sometimes incoherent with mathematical rules. But it shows that there are probability changes that cannot be represented in PUPDEL.[18] In this paper, I proposed GIPDEL and showed that these probability changes can be represented in GIPDEL. The modified version of this dilemma supported this opinion.

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[17] [[7]: 403].

[18] I argued in [10] and [11] on the relation between imaging and AGM and argued in [12] on the relation between imaging and the diachronic Dutch book argument.