The principle of reflection, nested sequents and proof-theoretic semantics

Hidenori Kurokawa
Kobe University

Abstract

In Sambin, Battilotti and Faggian (2000), they discuss a natural way of proof-theoretically introducing logical constants. Roughly, the idea is 1) to use an equivalence formula as a "definition" of a logical constant and 2) to derive ordinary operational rules in a cut-free sequent calculus for the logical constant only via minimal background assumptions. The idea works well for multiplicative and additive conjunction and disjunction. However, handling implication is not entirely unproblematic.

In this talk, we propose to extend the approach to naturally introduce implication, following Sambin et al.'s fundamental ideas but using a proof-theoretical framework ("nested sequent"), which is a little more general than traditional sequent calculi.

We argue that such a generalization is motivated both technically and philosophically. From a philosophical point of view, we first point out that there are some potential conceptual problems in intuitionistic logic and then we argue that, in order to analyze the issues, we are motivated in formulating strictly weaker logics than intuitionistic logic. When we do so, there are several reasons why traditional sequent calculi are not particularly convenient. From a technical point of view, we show that nested sequents can formulate a variety of non-classical logics in a uniform way. (We present both our own results and a survey of the literature.)

Time permitting, we also address two further issues related to the approach. One is the issue of how nested sequents can be compared with other generalizations of sequent calculi. The other is the question of how this approach is related to both old and recent issues on the meaning of logical constants that have been discussed in the literature of proof-theoretic semantics.