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A Non-Archimedean Model of Logic of Gradable Adjectives

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Degree semantics (e.g., [3]) and ordering semantics (e.g., [1]) are two most prevailing semantics intended to deal with such positive and comparative constructions of gradable adjectives as: (1) Mary is taller than Harry is. (2) Mary is exactly twice as tall as Harry is. (3) Mary is 170 cm tall. (4) Atlanta is hotter than Boston. (5) Atlanta is hotter than Boston by twice as much as Rio is hotter than Rome. (6) ♯ It is three times as hot in Atlanta as it is in Boston. (7) Mary is taller than Harry by more than Mary is more heavier than Harry. In degree semantics, the truth conditions of sentences are given by some functions from individuals to degrees that are semantic values of gradable adjectives. So it may give the truth conditions of (2), (3), (5) and (7) in an adequate way. However, those of (1) and (4) do not need such functions. On the other hand, in ordering semantics, ordering relations between individuals are taken primitive, and then degrees are derived from them. However, these derivations cannot be executed in a formal and general way. So ordering semantics may give the truth conditions of (1) and (4), but it cannot give those of (2), (3), (5) and (7) in an adequate way. Measurement-theoretic semantics is a general framework into which both degree semantics and ordering semantics can be incorporated. Lassiter [5] is one of the most influential literatures in measurement-theoretic semantics. Then, can Lassiter [5] give the truth conditions of (1)–(5) and (7), and explain the meaninglessness of (6)? We would like to consider it from a measurement-theoretic point of view. We classify scale types in terms of the class of admissible transformations ϕ . A scale is a triple $\langle U, V, f \rangle$ where U is an observed relational structure that is qualitative, V is a numerical relational structure that is quantitative, and f is a homomorphism from U into V. A is the domain of U and B is the domain of V. When the admissible transformations are all the functions $\phi : f(A) \to B$, where $f(A)$ is the range of f, of the form $\phi(x) := \alpha x; \alpha > 0$, ϕ is called a similarity transformation, and a scale with the similarity transformations as its class of admissible transformations is called a ratio scale. Length is an example of a ratio scale. When a scale is unique up to order, the admissible transformations are monotone increasing functions $\phi(x)$ satisfying the condition that $x \geq y$ iff $\phi(x) \geq \phi(y)$. Such scales are called ordinal scales. When the admissible transformations are all the functions $\phi : f(A) \to B$ of the form $\phi(x) := \alpha x + \beta$; $\alpha > 0$, ϕ is called a positive affine transformation, and a corresponding scale is called an interval scale. Temperature on the Fahrenheit scale and temperature on the Celsius scale are examples of interval scales. A scale is called a log-interval scale if the admissible transformations are functions of the form αx^{β} ; $\alpha, \beta > 0$. Next we classify measurement types. Suppose A is a set, R is a binary on A, $\bigcirc$ is a binary operation on A, D is a quaternary on A, and f is a real-valued function. Then we call such representation that aRb iff $f(a) > f(b)$ ordinal measurement (ORD), such representation that aRb iff $f(a) > f(b)$ and $f(a \bigcirc b) = f(a) + f(b)$ extensive measurement (EXT), such representation that abRst iff $f(a) - f(b) > f(s) - f(t)$ algebraic difference measurement (ALD), and such representation that abRst iff $f(a)/f(b) > g(s)/g(t)$ cross-modality measurement (CRM). Cantor [2] proves the following theorem: Fact 1 (Representation Theorem for Ordinal Measurement) There is a function $f : A \to R$ satisfying ORD for all $a, b \in A$ iff $U := \langle A, \succ \rangle$ is a structure satisfying Strict Weak Order. $\langle U, V, f \rangle$ is a ordinal scale, where $V := \langle R, > \rangle$.■ Roberts and Luce [7] prove the following theorem: Fact 2 (Representation Theorem for Extensive Measurement) There is a function $f : A \to R$ satisfying EXT for all $a, b \in A$ iff $U := \langle A, \succ, \bigcirc \rangle$ is a structure satisfying • Weak Associativity • Strict Weak Order • Monotonicity and • Archimedeanness. $\langle U, V, f \rangle$ is a ratio scale, where $V := \langle R, >, + \rangle$.■ Krantz et al. [4] prove the following theorem: Fact 3 (Representation Theorem for Algebraic Difference Measurement) If $U := \langle A, \succ_{(4)} \rangle$ is a structure satisfying • Strict Weak Order • Position Reversal • Weak Monotonicity • Solvability and • Archimedeanness, then there is a function $f : A \to R$ satisfying ALD for all $a, b, s, t \in A$. $\langle U, V, f \rangle$ is an interval scale, where $xy\Delta uv$ iff $x - y > u - v$ and $V := \langle R, \Delta \rangle$.■ The next corollary directly follows from Theorem 4 of [4, p. 165-166]: Fact 4 (Representation Theorem for Cross Modality Measurement) If $U := \langle A, \succ_{(4)} \rangle$ is a structure satisfying all conditions of Fact 3, then there are functions $f, g : A \to R$ so that CRM holds for all $a, b, s, t \in A$. $\langle U, V, f, g \rangle$ is a log-interval scale, where $xy\Delta uv$ iff $x/y > u/v$ and $V := \langle R, \Delta \rangle$.■ Not only degree and ordering semanticists but also Lassiter [5] presupposes that each gradable adjective can relate to only one scale type. We criticize this presupposition and propose the new idea that each gradable adjective can relate to several scale types according to such measurement types as ORD, EXT, ALD and CRM as follows: Lassiter [5, p.34] gives both the truth condition of (1) and that of (2) in terms of ratio scales, whereas he would give both the truth condition of (4) and that of (5) in terms of interval scales. However, when the measurement to determine the truth condition of (1) is based on ORD, a ratio scale is not necessary to justify the truth condition of (1), but an ordinal scale is sufficient to do so, When the measurement to determine the truth condition of (2) and that of (3) are based on EXT , a ratio scale is necessary to justify the truth condition of (2) and that of (3). When the measurement to determine the truth condition of (4) is based on ORD, an interval scale is not necessary to justify the truth condition of (4), but an ordinal scale is sufficient to do so. When the measurement to determine the truth condition of (5) is based on ALD, an interval scale is necessary to justify the truth condition of (5). When the measurement to determine the truth condition of (7) is based on CRM, a log-interval scale is necessary to justify the truth condition of (7). In order to cash this insight, we propose a new logic—Logic of Gradable Predicates (LGP) that has devices which can indicate the measurement-type of each well-formed formula where gradable adjectives occur. We define the language L_{LGP} of LGP: Definition 1 (Language). By use of the compactness theorem, Narens [6] proves the negative result: Fact 5 (Non-First-Order-Derivability of Archimedeanness) No set of first-order axioms imply Archimedeanness.■ Narens [6] proves the following theorem without Archimedeanness: Fact 6 (Non-Standard Representation Theorem for Extensive Measurement) Suppose that $_*R$ denotes the set of nonstandard reals (containing infinitesimals). Then there is a non-standard function $_*f : A \to {_*R}$ satisfying the left-to -right representation of EXT for all a, b ∈ A iff $U := \langle A, \succ, \bigcirc \rangle$ is a structure satisfying • Weak Associativity • Strict Weak Order and • Monotonicity. $\langle U, V, {_*f} \rangle$ is a ratio scale, where $V := \langle {_*R}, >, + \rangle$.■ We can prove the following theorem without Archimedeanness: Theorem 1 (Non-Standard Representation Theorem for Algebraic Difference Measurement) If $U := \langle A, \succ_{(4)} \rangle$ is a structure satisfying • Strict Weak Order • Position Reversal • Weak Monotonicity and • Solvability, then there is a non-standard function $_*f : A \to {_*R}$ satisfying the left-to-right representation of ALD for all a, b, s,t ∈ A. $\langle U, V, {_*f} \rangle$ is an interval scale, where $xy\Delta uv$ iff $x - y > u - v$ and $V := \langle {_*R}, \Delta \rangle$.■ We can prove the following theorem without Archimedeanness: Theorem 2 (Non-Standard Representation Theorem for Cross-Modality Measurement) f $U := \langle A, \succ_{(4)} \rangle$ is a structure satisfying all conditions of Theorem 1, then there are non-standard function $_*f, {_*g} : A \to {_*R}$ satisfying the left-to-right representation of CRM for all a, b, s,t ∈ A. $\langle U, V, {_*f}, {_*g} \rangle$ is a log-interval scale, where $xy\Delta uv$ iff $x/y > u/v$ and $V := \langle {_*R}, \Delta \rangle$.■ We define a non-Archimedean model M of LGP: Definition 2 (Non-Archimedean Model of LGP). We define satisfaction, truth and validity in LGP: Definition 3 (Satisfaction, Truth and Validity in LGP). By use of Facts 1 and 6, Theorems 1 and 2, and Definition 3, we can prove Proposition 1 (Truth Conditions in Terms of f ($_*f$)). By Proposition 1, we can provide all of (1)–(5) and (7) with their truth conditions and explain the meaninglessness of (6). Next we provide LGP with the proof system: Definition 4 (Proof System of LGP). Finally we prove the soundness and completeness theorems: Theorem 2 (Soundness of LGP) and Theorem 3 (Completeness of LGP).

（使用言語：日本語）

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