Measurement-Theoretic Foundations of First-Order Nominalism in Newtonian Gravitational Theory

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The aim of Field [1] is to defend nominalism which is a doctrine that there are no such abstract (mathematical) objects as numbers, functions, and sets. So he does not admit quantification over such mathematical objects. Because mathematical objects do not exist, mathematical theories are no bodies of true formulae. For Field, the one and only serious argument for the existence of mathematical objects is the Quine-Putnam Indispensability Argument: we cannot carry out inferences about the physical world without resort to physical theories that postulate mathematical objects. Field tries to undercut this argument and regards mathematical theories not as bodies of true formulae but as instruments for deriving nominalistically stated conclusions from nominalistically stated premises. The use of mathematical theories is considered to be good when they satisfy a kind of consistency condition that is the condition of being conservative in the sense that any nominalistically stated conclusions derivable with the help of mathematical theories are already derivable from the nominalistically stated premises only. Their usefulness consists in shortening our derivations. Abstract (mathematical) objects are useful because we can use them to formulate abstract counterparts of concrete (nominalistic) statements. Field considers the application of mathematics from a measurement-theoretic point of view. The representation theorem proves the existence of a structure-preserving mapping f from a *qualitative (comparative) structure* to a *quantitative (numerical) structure*. On the other hand, the uniqueness theorem specifies the transformation up to which f is unique. Field [1, pp.26–27] makes clear how f contributes to nominalism as follows:

- *First Step*: We can use f to ascend from concrete (nominalistically stated) premises to abstract counterparts.
- *Second Step*: By reasoning within a mathematical theory, we can prove abstract counterparts of further concrete (nominalistic) statements.
- *Third Step*: We can use f again to descend to the concrete statements of which they are abstract counterparts.

- So, the concrete conclusions so reached would always be obtainable without ascending to the abstract counterparts.

Field [1, ch.8] tries to nominalize Newtonian gravitational theory, which is the heart of Field [1]. Field provides a joint axiom system (*JAS*) that has the qualitative axioms for the representation and uniqueness theorems for the three functions:

1. a spatio-temporal coordinate function φ,

2. a mass-density function ρ, and

3. a gravitational potential function ψ.

Then Field shows that statements of Newtonian gravitational theory are expressible by using *JAS*. The qualitative axiom subsystem of *JAS* for the representation and uniqueness theorems for φ that is based on Szczerba and Tarski [4]'s axiomatization of affine geometry is *first-order axiomatizable*. Both the qualitative axiom subsystem of *JAS* for the representation and uniqueness theorems for ρ and that for ψ are for *difference measurement*, from a measurement theoretic point of view . According to Field [1, p.38], only one second-order axiom for the representation theorem for difference measurement is the Dedekind completeness axiom that implies Archimedeanness: for any positive number x, no matter how small, and for any number y, no matter how large, there exists an integer n such that $nx \geq y$. This axiom quantifies over non-empty sets of points. Field [1, p.92] remarks on the following two respects in which he has overstepped the bounds of first-order logic into second-order logic:

1. the complete logic of Goodmanian sums,

2. the binary quantifier "there are only finitely many".

Then Field [1, pp.92–93] argues that we do not really need the latter in addition to the former, for the former is sufficient to give us the cardinality comparisons and the representation theorems. Field [1, ch.9] tries to nominalize the Dedekind completeness axiom by identifying a non-empty set of points with a Goodmanian sum of points. From this viewpoint, we should rewrite the fourth bullet item as follows:

- So, the concrete conclusions so reached would always be obtainable without ascending to the abstract counterparts with the help of the complete logic of Goodmanian sums.

This way of identification may be worth studying, but requires many complicated axioms including second-order ones [1, ch,9]. In this talk we would like to investigate the possibility of *first-order nominalism* in terms of the following representation and uniqueness theorems: The aim of this talk is that, in order to investigate the possibility of first-order nominalism, introducing $\mathbb{R}' := \mathbb{R} \cup \{-\infty, +\infty\}$, we prove new non-standard representation and uniqueness theorems for difference measurement without Archimedeanness, in other words, with only first-order axioms by making use of [2, 3]. As well as [2, 3], our representation theorem does not guarantee the existence of such f that if $a \succ b$ then $f(a) > f(b)$ which can imply that if $f(a) = f(b)$ then $a \sim b$ but guarantees the existence of such f that if $f(a) > f(b)$ then $a \succ b$ which can imply that if $a \sim b$ then $f(a) = f(b)$. So, about *equalities*, our representation theorem does not justify the *Third Step* above but justifies the *First Step* above. About *inequalities*, our representation theorem does not justify the *First Step* but justifies the *Third Step*. This point can obtain the following interpretative support: On the basis of Mundy [2, pp.388–389], where such qualitative relations as $\succ$ and $\sim$ are not *observational (empirical)* in the standard sense in measurement theory but *theoretical*, we can argue as follows:

- about Equalities
  - $f(a) = f(b)$ simply means a failure to detect any actual difference, which is no guarantee that finer observations will detect no difference. So $f(a) = f(b)$ does not entail $a \sim b$.
  - $a \sim b$ means exact theoretical equivalence and hence equivalence should hold for any measuring process. So $a \sim b$ entails $f(a) = f(b)$.
- about Inequalities
  - On the other hand, $a \succ b$ means a theoretical difference that may not manifest itself in our observations, and hence not in our numerical scale assignments. So $a \succ b$ does not entail $f(a) > f(b)$.
  - $f(a) > f(b)$ means a detection of an actual difference, and the actual difference will also be detectable by a finer observation. So $f(a) > f(b)$ entails $a \succ b$.

References

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[4] L. W. Szczerba and A. Tarski. Metamathematical properties of some affine geometries. In Y. Bar-Hillel, editor, *Proceedings of the 1964 International Congress for Logic, Methodology and Philosophy of Science*, pages 166–178. North-Holland, Amsterdam, 1965.