Additively-Semiordered-Qualitative-Conditional-Probability-Theoretic Foundations of Logic of Inexact Knowledge

鈴木　聡（Satoru Suzuki）

駒澤大学総合教育研究部非常勤講師

Williamson (1994, p.21) investigates inexact knowledge. He (ibid, pp.218—220) presents a version of sorites paradox that we call the paradox of the knowledge on a crowd. He (ibid, pp.222—223) goes on to give an analysis that the culprit of this paradox is the KK principle. He (ibid, p.226) relates inexact knowledge to a margin for error and argues that the width of a margin for error depends on the cognitive capacities. He (ibid, p.227) states that each case of inexact knowledge is governed by a margin for error principle saying that `φ' is true in all cases similar to cases in which `It is known that φ' is true. He (ibid, p.227) characterizes a margin for error meta-principle in such a way that inexact knowledge is related to a margin for error principle: Where knowledge is inexact, some margin for error principle holds. Moreover, he (ibid, p.227—228) relates the KK principle to inexact knowledge, a margin for error principle, and a margin for error meta-principle. He (ibid, pp.237--238) classifies indiscriminability into two types: direct indiscriminability and indirect one, and then relates them: Call two things indirectly indiscriminable in a certain respect just in case they are directly indiscriminable in that respect from exactly the same things. He observes that direct indiscriminability is a non-transitive relation, while `indirect indiscriminability is by definition a transitive relation' (ibid, p.238) and that `indirect discrimination is not a genuinely cognitive form of discrimination at all' (ibid, p.240). He (ibid, p.241) argues that with the KK principle, a sorites paradox of knowledge in which indiscriminability (being the same as) occurs would be forthcoming. We call this paradox the paradox of the knowledge on the height of a tree. He (ibid, p.242) points out that `the KK principle fails because the indiscriminability of worlds is non-transitive. The example began with the non-transitive indiscriminability of days in the height of the tree, and moved on to a similar phenomenon for worlds. … The indiscriminability of the objects is equivalent to the indiscriminability of the corresponding worlds, and therefore to their accessibility'. In this talk, we discuss Williamson's arguments above from a general point of view. The standard models of social sciences are based on global rationality that requires an optimising behavior. But according to Simon (1982), cognitive and information-processing constrains on the capabilities of agents, together with the complexity of their environment, render an optimising

behavior an unattainable ideal. Simon dismisses the idea that agents should exhibit global rationality and suggests that they in fact exhibit bounded rationality that allows a satisficing behavior. If an agent has only a limited ability of discrimination, he may be considered to be only boundedly rational. The margin for error principle can be regarded as an instance of bounded rationality. From a psychophysical point of view, we consider a margin for error. The psychophysicist Fechner (1860) explains this limited ability by the concept of a threshold of discrimination, that is, just noticeable difference (JND). Given a measure function f that an examiner could assign to a boundedly-rational examinee for an object a, its JND δ is the lowest intensity increment such that f(a)+δ is recognized to be higher than f(a) by the examinee. JND can be considered to be a psychophysical counterpart of a margin for error. We can consider a JND from a probabilistic point of view. Domotor (1969, pp.90—98) introduces the concept of additively-semiordered qualitative conditional probability that can provide a qualitatively-probabilistic counterpart of a JND. The aim of this talk is to propose a new version of complete logic---Logic of Inexact Knowledge (LIK)---the model of which can reflect Williamson's arguments above in the sense that it has the following seven features:

(1) This model based on additively-semiordered qualitative conditional probability that is a qualitatively-probabilistic counterpart of a JND which is a psychophysical counterpart of a margin for error can reflect the essence of inexact knowledge.
(2) By using this model, we can prove a margin for error principle.
(3) The width of a margin for error (JND) depends on the cognitive capacities.
(4) A direct indiscriminability relation is a non-transitive relation.
(5) Inexact knowledge is defined in terms of this direct indiscriminability relation.
(6) LIK has so rich expressive power as to formalize such sorites paradoxes of knowledge as the above-mentioned paradoxes of the knowledge on a crowd and on the height of a tree.
(7) Because the KK principle is not valid in LIK, these sorites paradoxes of knowledge do not arise.（使用言語：日本語）

References

・Domotor, Z.: Probabilistic Relational Structures and Their Applications. Technical Report No. 144, Institute for Mathematical Studies in the Social Sciences, Stanford University (1969)
・Fechner, G.T.: Elemente der Psychophysik. Breitkopf und Hartel, Leipzig (1860)
・Simon, H.A.: Models of Bounded Rationality, Vol l. The MIT Press, Cambridge, Mass. (1982)
・Williamson, T.: Vagueness. Routledge, London (1994)