Bounds of Full-Blooded First-Order Nominalism

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The aim of Field (2016) is to defend nominalism which is a doctrine that there are no such abstract (mathematical) objects as numbers and functions. So he does not admit quantification over such mathematical objects. Because mathematical objects do not exist, mathematical theories are no bodies of true formulae. For Field, the one and only serious argument for the existence of mathematical objects is the Quine-Putnam Indispensability Argument: we cannot carry out inferences about the physical world without resort to physical theories that postulate mathematical objects. Field tries to undercut this argument and regards mathematical theories not as bodies of true formulae but as instruments for deriving nominalistically stated conclusions from nominalistically stated premises. The use of mathematical theories is considered to be good when they satisfy conservativeness: any nominalistically stated conclusions derivable with the help of mathematical theories are already derivable from the nominalistically stated premises only. Their usefulness consists in shortening our derivations. Abstract (mathematical) objects are useful because we can use them to formulate abstract counterparts of concrete (nominalistic) statements. Field considers application of mathematics from a measurement-theoretic point of view: (1) The representation theorem proves the existence of a homomorphism (structure-preserving mapping) f from a concrete (qualitative, comparative) structure to a mathematical (quantitative, numerical) structure. (2) The uniqueness theorem specifies the transformation up to which f is unique. Field (2016, pp.26--27) makes clear how f contributes to nominalism as follows: First Step: We can use f to ascend from concrete (nominalistically stated) premises to abstract counterparts. Second Step: By reasoning within a mathematical theory, we can prove the abstract counterparts of further concrete (nominalistic) statements. Third Step: We can use f again to descend to the concrete statements of which they are the abstract counterparts. By conservativeness, the concrete conclusions so reached would always be obtainable without ascending to the abstract counterparts. Field (2016, ch.8) tries to nominalize Newtonian gravitational theory, which is the heart of Field (2016). Field provides a qualitative (concrete) joint axiom system (JAS) that has the qualitative axioms for the representation and uniqueness theorems for the three functions: (1) a spatio-temporal coordinate function φ, (2) a mass-density function ρ, and (3) a gravitational potential function ψ. Then Field shows that statements of Newtonian gravitational theory are expressible by using JAS. The qualitative axiom subsystem of JAS for the representation and uniqueness theorems for φ that is based on Szczerba and Tarski (1965)'s axiomatization of affine geometry is first-order axiomatizable. Both the qualitative axiom subsystem of JAS for the representation and uniqueness theorems for ρ and that for ψ are of algebraic-difference measurement in measurement theory. According to Field (2016, p.38), only one second-order axiom for the representation theorem of algebraic-difference measurement is the Dedekind completeness axiom that implies Archiedeanness: for any positive number x, no matter how small, and for any number y, no matter how large, there exists an integer n such that $nx \geqq y$. This axiom quantifies over an integer n (non-empty sets of points). Field (2016, p.92) remarks on the following two respects in which he has overstepped the bounds of first-order logic into second-order logic: (1) merological sum, (2) the binary quantifier ``there are only finitely many''. Field (2016, ch.9) tries to nominalize the Dedekind completeness axiom by identifying a non-empty set of points with a mereological sum of points. This way of identification faces the difficulty of requiring many very complicated axioms including even second-order ones (ch.9) that are not nominalistic. The aim of this talk is to investigate the bounds of the full-blooded first-order nominalism: In order to determine which steps of the First and Third Steps above of Field's mixed-blooded nominalism (including even second-order axioms that are not nominalistic) the full-blooded first-order nominalism can guarantee, we prove the following two pairs of brand-new representation and uniqueness theorems for difference measurement without Archimedeanness, in other words, with only respective sets of first-order axioms for a representation theorem that both quaternary qualitative relation of gravitational potential and that of mass density satisfy. Our representation and uniqueness theorems introduce the hyperreals *R and make use of Narens (1974) in which the representation and uniqueness theorems on *R for conjoint measurement are proven. We end in the following dilemma in nominalism: If, like Field, we keep to the mixed-blooded nominalism, it can guarantee both the First and Third Steps above. But it requires second-order axioms that are not nominalistic. If we keep to the full-blooded first-order (non-Archimedean) nominalism, by means of our first and second pairs of representation and uniqueness theorems, it can guarantee the First Step, but it cannot guarantee the Third Step. （使用言語：日本語）

References

・ H.H. Field. Science Without Numbers, Second Edition. Oxford University Press, Oxford, 2016.

・ L. Narens. Minimal Conditions for Additive Conjoint Measurement and Qualitative Probability. Journal of Mathematical Psychology, 11: 404--430, 1974.

・ L.W. Szczerba and A. Tarski. Metamathematical Properties of Some Affine Geometries. In Y. Bar-Hillel (ed.), Proceedings of the 1964 International Congress for Logic, Methodology and Philosophy of Science, pp.166--178, North-Holland, Amsterdam, 1965.